Technical Report CCSR-89-3

Characterization of Complex Systems By Aperiodic Driving Forces *

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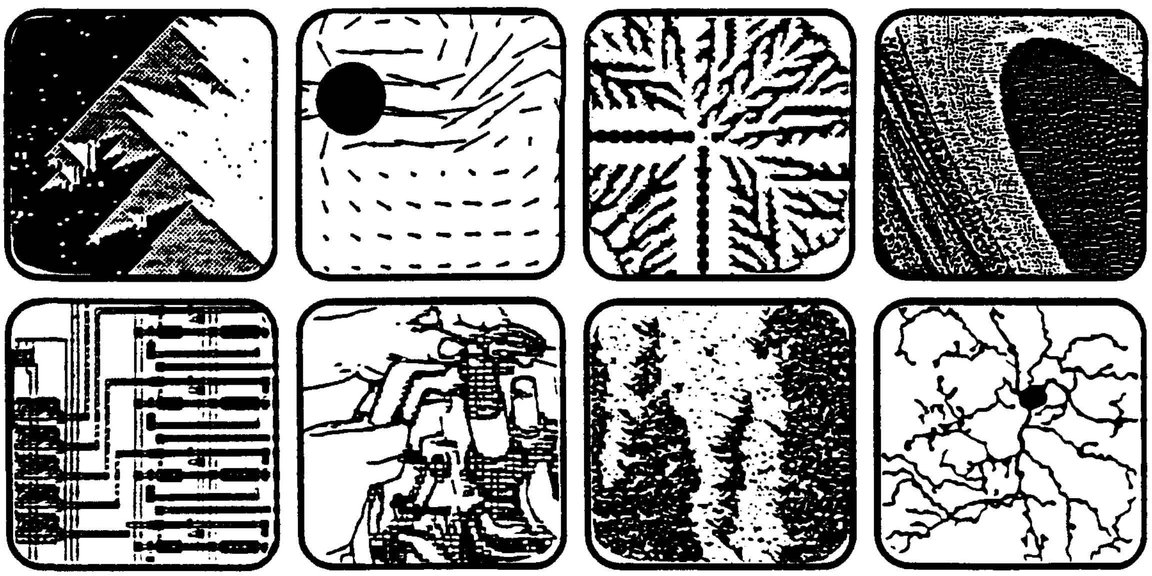

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Abstract. The response of a complex system is usually very complicated if it is perturbed by a sinusiodal driving force. We show, however, that for every complex system there is a special aperiodic driving force which produces a simple response. This special driving force is related to a certain nonlinear differential equation. We propose to use the parameters of this differential equation to describe the complexity of the system.

INTRODUCTION

Generalized dimensions, entropies, Lyapounov exponents [1] and approximations of the flow vector field [2,3] are used to describe the periodic and chaotic dynamics of nonlinear experimental systems. In addition to the passive observation of a nonlinear oscillator and the description of the measured time series using statistical quantities, it is possible to characterize a nonlinear oscillator by an active method, namely by determining its response to specific driving forces [4]. The output of the active method is usually the numerical values of the parameters and the dimension of a differential equation or a map which models the dynamics of the system. These parameters can be used in order to define classes of complexity [5]. Active methods, like nonlinear resonance spectroscopy, are superior to passive methods because of three reasons: First, a measurement of parameters of a system by an active method can be done in the region of the state space of the system that is of the most physical interest, in contrast to passive methods, which have access only to those regions of the state space which are close to the attractors. Second, the model can be used to control the dynamics of complex systems [6,7]. Third, the active method is especially superior in those cases where the the experimental system is a set of identical, weakly coupled oscillators behaving incoherently. If one gets without a driving force a compound signal

*To appear in *Quantitative Measures of Complex Dynamical Systems*, edited by N.B. Abraham, Plenum Press, New York, 1989

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of all oscillators, small and complicated due to interference, a strong response emerges at resonance. At resonance every single oscillator is forced into coherent oscillation, synchronized by the driving force. In the next section we show that the response of a nonlinear oscillator to certain aperiodic driving forces can be several orders of magnitude larger than the response to sinusoidal perturbations and how to use this response in order to find a model of the dynamics.

RESONANT PERTURBATIONS OF NONLINEAR OSCILLATORS

In order to investigate resonant driving forces we consider in the following a damped or conservative oscillation in a nonlinear Potential $V(y)$:

$$\ddot{y} + \eta \dot{y} + \frac{\partial V(y, \vec{p})}{\partial y} = F(t) \tag{0.1}$$

where $\eta \geq 0$ is a friction constant and $\vec{p}$ are the parameters of the potential. In order to calculate resonant driving forces, we integrate a goal equation [7]:

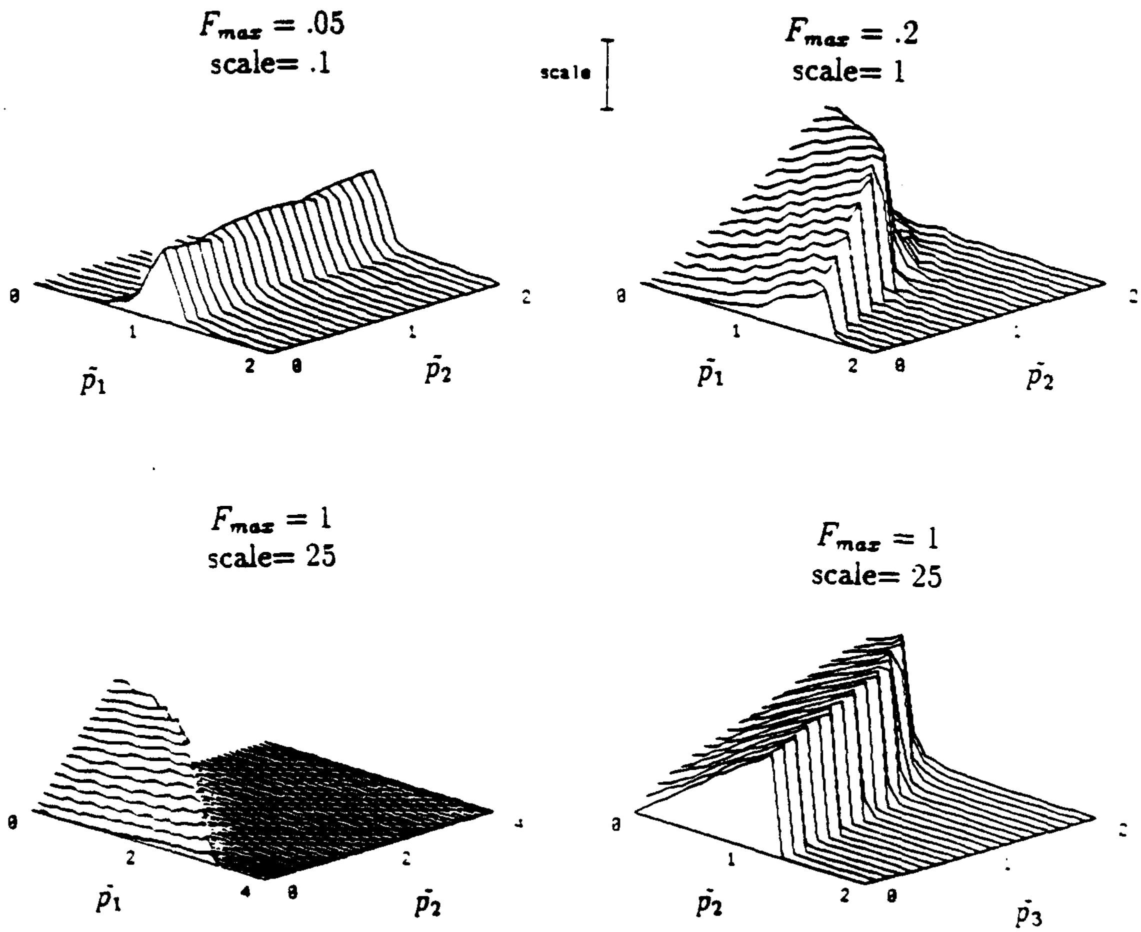

Fig. 1 illustrates the final energy $E\,(T = 50) = .5\dot{y}^2 + V\,(y, \vec{p})$ versus the parameters of the model of the potential $V\left(x, \vec{\tilde{p}}\right) = .5\tilde{p_1}x^2 + .25\tilde{p_2}x^4 + .01\tilde{p_3}x^6$, where $p_1 = p_2 = p_3 = 1$ are the parameters of the experimental system and where the magnitude of the driving force has various values. The initial conditions are $y = \dot{y} = 0$ and $x = \dot{x} \approx .02$. At resonance, i.e. for $\tilde{p_1} = \tilde{p_2} = \tilde{p_3} = 1$ the final energy is more than one order of magnitude larger than the largest final energy due to a sinusoidal perturbation with the same amplitude ($F_{max} = 1$).

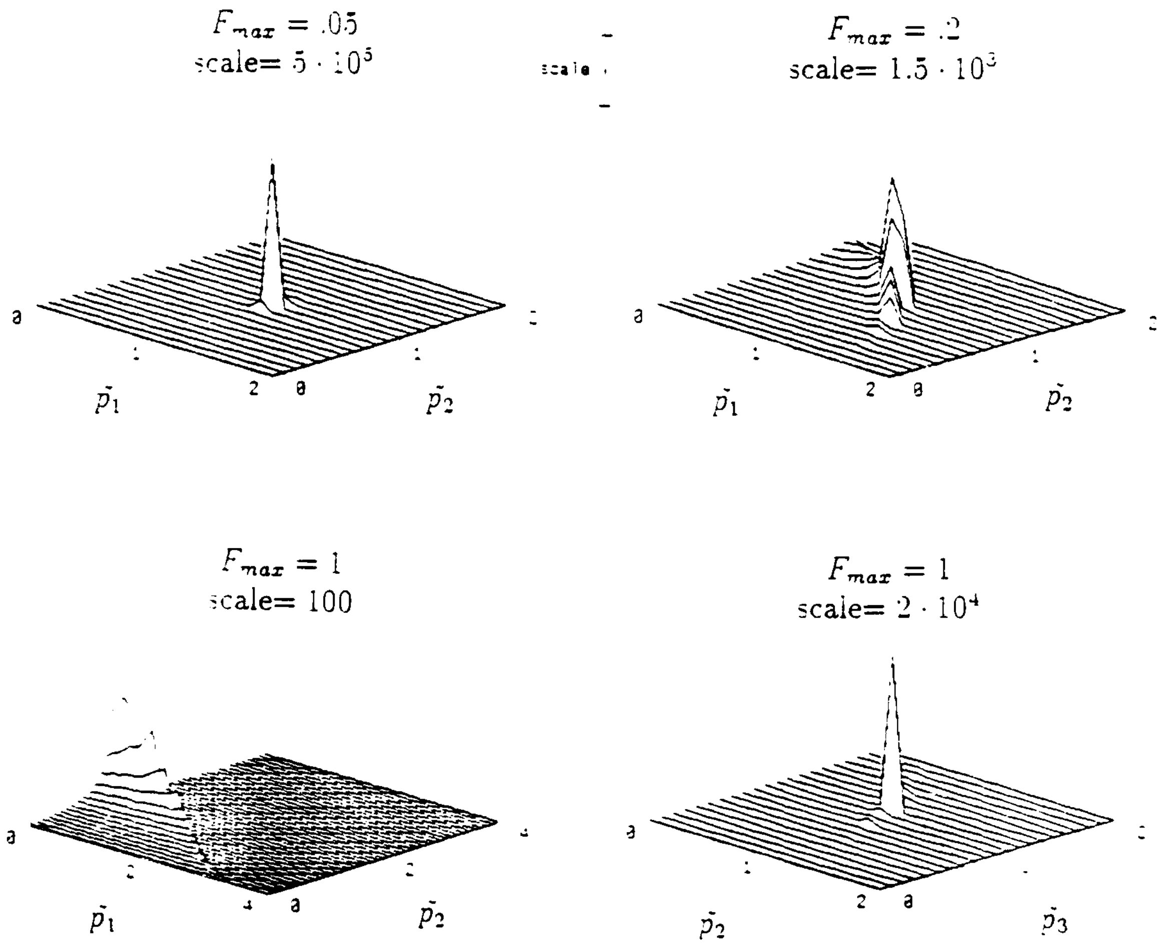

Fig. 2 illustrates the ratio between the transfered and the reflected energy versus the parameters of the model of the potential $V\left(x, \vec{\tilde{p}}\right) = .5\tilde{p_1}x^2 + .25\tilde{p_2}x^4 + .01\tilde{p_3}x^6$, where $p_1 = p_2 = p_3 = 1$ are the parameters of the experimental system and where the magnitude of the driving force has various values. The initial conditions are $y = \dot{y} = 0$ and $x = \dot{x} \approx .02$.

$$\ddot{x} + \tilde{\eta}(t)\,\dot{x} + \frac{\partial V\left(x, \vec{\tilde{p}}\right)}{\partial x} = 0 \tag{0.2}$$

where $\tilde{\eta}(t) = \eta - \frac{F_{max}}{(2E)^{.5}}$. F_{max} estimates the amplitude of the driving force defined by $F(t) = (\eta - \tilde{\eta}(t))\,\dot{x}$. Due to the special choice of $\tilde{\eta}(t)$ the amplitude of the driving force is approximately constant. If $\vec{\tilde{p}} = \vec{p}\; y(t) = x(t)$ is a solution of Eq. (0.1). For a large variety of systems this solution is stable [6]. For $y(t) = x(t)$ the energy transfer $\Delta E = \int_0^T F(t)\,\dot{y}dt$ is positive for all T and the reflected energy $E_r = \int_0^T F(t)\,\dot{y}\Theta(-F(t)\,\dot{y})\,dt$ is zero, which is its smallest value. Θ is the Heavyside step function. For $\vec{\tilde{p}} \neq \vec{p}$ the reflected energy is non zero and the energy transfer is much smaller (see Fig.1 and Fig.2).

NONLINEAR RESONANCE SPECTROSCOPY

The largest response emerges when the parameters of the model coincide with the true parameters of the system (Figs. 1,2). Therefore the parameters of the experimental system can be found by a systematic search for the largest response. This method is equivalent to the usual linear resonance spectroscopy for $\tilde{p}_2 = 0$. In the linear case the driving forces are sinusoidal perturbations and the corresponding resonance curves result. They represent a cut through the shoulder of the nonlinear resonance peak in the background. For large driving forces the maximum amplitude of the linear resonance curves is usually several orders of magnitude smaller than the maximum amplitude of the nonlinear resonance curve. A general problem of finding a model of a nonlinear system is the large number of parameters which might be necessary. A systematic scan in the high dimensional parameter space in order to find the maximum response is often too time consuming. Therefore we propose the following systematic search: When a small driving force is applied, the response is sensitive only to the linear parameter of the model (see Fig. 1a). Therefore a small driving force can be used to determine this parameter. If a moderate driving force is applied the response is sensitive to the parameter of the next order, but is still insensitive to higher order terms. By a systematic increase of the magnitude of the aperiodic driving force the coefficients of a Taylor series of the flow vector field of the model can be determined step by step.

This work was supported in part by the grant N0001 , 88-K-0293 from the ONR and by the grant NSF-PHY 86-58062

References

[1] see e.g. J.P. Eckmann and D. Ruelle, "Ergodic Theory of Chaos and Strange Attractors", *Rev.Mod.Phys.* **57** 617 (1985).

[2] J. Cremers and A. Hübler, "Construction of Differential Equations from Experimental Data", *Z.Naturforsch* **42a** 797 (1987); J.P. Crutchfield and B.S. McNamara, "Equations of Motions from Data Series", *Complex Systems*, **1** 417 (1987).

[3] J.D. Farmer and J.J. Sidorowich, "Predicting Chaotic Time Series", *Phys.Rev.Lett.*, **59** 845 (1987).

[4] G. Reiser, A. Hübler, and E. Lüscher, "Algorithm for the Determination of the resonances of Anharmonic Damped Oscillators", *Z.Naturforsch*, **42a** 803 (1987).

[5] R. Thom, *Structural Stability and Morphogenisis*, (W.A. Benjamin, Reading, Mass. 1975).

[6] A. Hübler and E. Lüscher, "Resonant Stimulation and Control of Nonlinear Oscillators", *Naturwissenschaften*, **76** 67(1989).

[7] A. Hübler, "Adaptive Control of Chaotic Systems", to appear in *Helv.Phys.Acta*, **61** (1989).